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$\mathbf{SU}(3)_L \otimes \mathbf{U}(1)_N$ and $\mathbf{SU}(4)_L \otimes \mathbf{U}(1)_N$ gauge models with right-handed neutrinos

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Abstract

Pisano and Pleitez have introduced an interesting $\mathrm{SU}(3)_C \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_N$ gauge model which has the property that gauge anomaly cancellation requires the number of generations to be a multiple of 3. We consider generalizing that model to incorporate right-handed neutrinos. We find that there exists a non-trivial generalization of the Pisano-Pleitez model with right-handed neutrinos which is actually simpler than the original model in that symmetry breaking can be achieved with just three $\mathrm{SU}(3)_L$ triplets (rather than 3 $\mathrm{SU}(3)_L$ triplets and a sextet). We also consider a gauge model based on $\mathrm{SU}(3)_C \otimes \mathrm{SU}(4)_L \otimes \mathrm{U}(1)_N$ symmetry. Both of these new models also have the feature that the anomalies cancel only when the number of generations is divisible by 3.

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The standard model of electroweak interactions and QCD has proven extremely successful. In the last decade no new elementary particles or interactions have been discovered beyond the expected confirmation of the W and Z bosons of the standard model. To go beyond the standard model, we should look for interesting ideas until novel events are discovered.

Pisano and Pleitez [1] have proposed an interesting model based on the gauge group

$$\mathrm{SU}(3)_c \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_N \tag{1}$$

(for further work on this model see Ref. [2,3].) This model has the interesting feature that each generation of fermions is anomalous, but that with three generations the anomalies cancelled.

In this paper we point out that if right-handed neutrinos are included then there is an alternative $\mathrm{SU}(3)_L \otimes \mathrm{U}(1)_N$ gauge model. This alternative model is actually simpler than the Pisano Pleitez model because it turns out that less Higgs mulitplets are needed in-order to allow the fermions to gain masses and to break the gauge symmetry. This alternative model also has the interesting property that anomalies only cancel when all three generations are included. This alternative model allows for Dirac neutrino masses. We will also discuss a $\mathrm{SU}(3)_C \otimes \mathrm{SU}(4)_L \otimes \mathrm{U}(1)_N$ gauge model which also includes right-handed neutrinos and also requires 3 generations to cancel the anomalies.

We start by briefly reviewing the Pisano-Pleitez model. In that model the three lepton generations transform under the gauge symmetry, Eq.(1) as

$$f_L^a = \begin{pmatrix} \nu_L^a \\ e_L^a \\ (e_R^c)^a \end{pmatrix} \sim (1, 3, 0), \tag{2}$$

where $a = 1, 2, 3$ is the generation index.

Two of the three quark generations transform identically and one generation, transforms in a different representation of $\mathrm{SU}(3)_C \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_N$:

$$\begin{aligned}
&Q_{1L} = \begin{pmatrix} u_1 \\ d_1 \\ J_1 \end{pmatrix}_L \sim (3, 3, 2/3), \\
&u_{1R} \sim (3, 1, 2/3),\ d_{1R} \sim (3, 1, -1/3),\ J_{1R} \sim (3, 1, 5/3), \\
&Q_{iL} = \begin{pmatrix} d_i \\ u_i \\ J_i \end{pmatrix}_L \sim (3, \bar{3}, -1/3), \\
&u_{iR} \sim (3, 1, 2/3),\ d_{iR} \sim (3, 1, -1/3), J_{iR} \sim (3, 1, -4/3),
\end{aligned} \tag{3}$$

where $i = 2, 3$.

Symmetry breaking and fermion mass generation can be achieved by three scalar $\mathrm{SU}(3)_L$ triplets and a sextet. For these details the reader can see Ref.[1].

If right-handed neutrinos are included then we can add either three $\mathrm{SU}(3)_L \otimes \mathrm{U}(1)_N$ singlets $\nu_R^a \sim (1,1,0)$ (which is a trivial generalization of the Pisano-Pleitez model), or we can try to modify the quantum numbers of the fermions such that ν_R replaces e_R as the third component of the lepton triplets. We will show that this latter case is possible and it leads to a $\mathrm{SU}(3)_L$ model which is simpler than the Pisano-Pleitez model with the same nice features.

The gauge quantum numbers of the fermions are as follows: The leptons consist of:

$$f_L^a = \begin{pmatrix} \nu_L^a \\ e_L^a \\ (\nu_R^c)^a \end{pmatrix} \sim (1,3,-1/3), \;\; e_R^a \sim (1,1,-1), \tag{4}$$

where $a = 1, ..., 3$. As in the case of the Pisano-Pleitez model, two of the quark generations transform identically, and one generation transforms differently:

$$\begin{aligned}
&Q_{1L} = \begin{pmatrix} u_{1L} \\ d_{1L} \\ u'_{1L} \end{pmatrix} \sim (3,3,1/3), \\
&u_{1R} \sim (3,1,2/3), \;\; d_{1R} \sim (3,1,-1/3), \;\; u'_{1R} \sim (3,1,2/3), \\
&Q_{iL} = \begin{pmatrix} d_{iL} \\ u_{iL} \\ d'_{iL} \end{pmatrix} \sim (3,\bar{3},0), \\
&u_{iR} \sim (3,1,2/3), \;\; d_{iR} \sim (3,1,-1/3), \;\; d'_{iR} \sim (3,1,-1/3),
\end{aligned} \tag{5}$$

where $i = 2, 3$. It is straightforward to check that all gauge anomalies cancel with the above choice of gauge quantum numbers. As in the Pisano-Pleitez model, each generation is anomalous but with all three generations the anomalies cancel. Symmetry breaking and fermion mass generation can be achieved with just three $\mathrm{SU}(3)_L$ Higgs triplets. We define them by their Yukawa Lagrangians as follows:

$$\mathcal{L}_{yuk}^{\chi} = \lambda_1 \bar{Q}_{1L} u'_{1R} \chi + \lambda_{2ij} \bar{Q}_{iL} d'_{jR} \chi^* + H.c., \tag{6}$$

where $\chi \sim (1,3,-1/3)$ and if χ gets the vacuum expectation value (VEV):

$$\langle \chi \rangle = \begin{pmatrix} 0 \\ 0 \\ w \end{pmatrix}, \tag{7}$$

then the exotic $2/3$ and $-1/3$ quarks gain masses and the gauge symmetry is broken to the standard model gauge symmetry:

$$\begin{array}{c} SU(3)_c \otimes SU(3)_L \otimes U(1)_N \\ \downarrow \langle \chi \rangle \\ SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \end{array} \tag{8}$$

where $Y = 2N - \sqrt{3}\lambda_8/3$ ($\lambda_8 = diag(1,1,-2)/\sqrt{3}$), is the combination of N and λ_8 which annihilates the VEV (i.e. $Y\langle\chi\rangle = 0$). Note that Y is identical to the standard hypercharge of the standard model. Electroweak symmetry breaking and ordinary fermion mass generation are achieved with two SU$(3)_L$ triplets ρ, η which we define through their Yukawa Lagrangians as follows:

$$\mathcal{L}^{\rho}_{Yuk} = \lambda_{1a}\bar{Q}_{1L}d_{aR}\rho + \lambda_{2ia}\bar{Q}_{iL}u_{aR}\rho^* + G_{ab}\bar{f}^a_L(f^b_L)^c\rho^* + G'_{ab}\bar{f}^a_L e^b_R\rho + H.c.,$$
$$\mathcal{L}^{\eta}_{yuk} = \lambda_{3a}\bar{Q}_{1L}u_{aR}\eta + \lambda_{4ia}\bar{Q}_{iL}d_{aR}\eta^* + H.c., \tag{9}$$

where $\rho \sim (1,3,2/3)$, $\eta \sim (1,3,-1/3)$ and we require the vacuum structure:

$$\langle\rho\rangle = \begin{pmatrix} 0 \\ u \\ 0 \end{pmatrix}, \ \langle\eta\rangle = \begin{pmatrix} v \\ 0 \\ 0 \end{pmatrix}. \tag{10}$$

The VEV $\langle\rho\rangle$ will generate masses for the three charged leptons and two of the neutrinos will gain degenerate Dirac masses (with one necessarily massless) and two up-type quarks and one down-type quark will also gain masses while the VEV $\langle\eta\rangle$ will generate masses for the remaining quarks. The VEVS $\langle\rho\rangle, \langle\eta\rangle$ also give the electroweak gauge bosons masses and results in the symmetry breaking:

$$\begin{array}{c} SU(3)_c \otimes SU(3)_L \otimes U(1)_N \\ \downarrow \langle\chi\rangle \\ SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \\ \downarrow \langle\rho\rangle, \langle\eta\rangle \\ SU(3)_c \otimes U(1)_Q \end{array} \tag{11}$$

Note that because $\langle\rho\rangle, \langle\eta\rangle$ transform as part of a $Y = 1$, SU$(2)_L$ doublet (under the SU$(2)_L \otimes$ U$(1)_Y$ subgroup of SU$(3)_L \otimes$ U$(1)_N$) the correct W, Z mass relation ensues, and the model essentially reduces to the standard model provided $\langle\chi\rangle \gg \langle\eta\rangle, \langle\rho\rangle$.

An important phenomenological difference between this model and the Pisano-Pleitez model is that the exotic quarks have electric charges $2/3$ and $-1/3$. (in the Pisano-Pleitez model the exotic quarks had electric charges $5/3$ and $-4/3$). A consequence of this is that the exotic quarks can mix with the ordinary ones. Indeed, in eq.(9), we can have extra terms obtained by replacing d_{aR}, u_{aR} with d'_{iR}, u'_{1R}. One important consequence of this type of mixing is that small Flavour changing neutral currents (FCNCs) will be induced due to the breakdown of the GIM mechanism (one can easily check however that as $\langle\chi\rangle$ goes to infinity these induced FCNCs go to zero). This type of situation has been discussed previously and bounds on the mixing strengths can be obtained from the experimental non-observation of FCNCs beyond those predicted by the standard model [4].

We now turn to another possibility beyond the standard model. This extension of the standard model is based on SU$(3)_C \otimes$ SU$(4)_L \otimes$ U$(1)_N$ gauge group [3]. In the

$\mathrm{SU}(3)_C \otimes \mathrm{SU}(4)_L \otimes \mathrm{U}(1)_N$ model, the three lepton generations transform under the gauge symmetry as

$$f_L^a = \begin{pmatrix} \nu_L^a \\ e_L^a \\ (\nu_R^c)^a \\ (e_R^c)^a \end{pmatrix}_L \sim (1,4,0), \tag{12}$$

where a=1,2,3 is a generation index. In the quark sector, two of the three quark generation transform identically and one generation transforms in a different representation of $\mathrm{SU}(4)_L \otimes \mathrm{U}(1)_N$. The quarks have the following representation under the $\mathrm{SU}(3)_C \otimes \mathrm{SU}(4)_L \otimes \mathrm{U}(1)_N$ gauge group:

$$Q_{1L} = \begin{pmatrix} u_1 \\ d_1 \\ u_1^{'} \\ J_1 \end{pmatrix}_L \sim (3,4,2/3),$$

$$u_{1R} \sim (3,1,2/3), d_{1R} \sim (3,1,-1/3), u_{1R}^{'} \sim (3,1,2/3), J_{1R} \sim (3,1,5/3),$$

$$Q_{iL} = \begin{pmatrix} d_i \\ u_i \\ d_i^{'} \\ J_i \end{pmatrix}_L \sim (3,\bar{4},-1/3),$$

$$u_{iR} \sim (3,1,2/3), d_{iR} \sim (3,1,-1/3), d_{iR}^{'} \sim (3,1,2/3), J_{iR} \sim (3,1,-4/3), \tag{13}$$

where $i = 2,3$. All gauge anomalies cancel in this theory. As discussed in Ref.[3] this type of construction is only anomaly free when the number of generations is divisible by 3. In the fermion representations we have added right-handed neutrinos and the exotic quarks $u_1^{'}, d_{2,3}^{'}, J_{1,2,3}$.

We now discuss symmetry breaking in this model. We introduce the Higgs field

$$\chi_1 \sim (1,4,-1), \tag{14}$$

which couples via the Yukawa Lagrangian

$$\mathcal{L}_{Yuk}^{\chi_1} = \lambda_1 \bar{Q}_{1L} J_{1R} \chi_1 + \lambda_{1ij} \bar{Q}_{iL} J_{jR} \chi_1^* + \text{H.c.}, \tag{15}$$

where $i,j \; = \; 2,3$. If χ_1 gets the VEV:

$$\langle \chi_1 \rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ w_1 \end{pmatrix}, \tag{16}$$

then the exotic charged 5/3 and -4/3 quarks ($J_{1,2,3}$) gain masses. In order for $u_1^{'}$ and $d_{2,3}^{'}$ to gain masses, we introduce the Higgs field

$$\chi_2 \sim (1,4,0), \tag{17}$$

which couples via the Yukawa Lagrangian

$$\mathcal{L}^{\chi_2}_{Yuk} = \lambda_2 \bar{Q}_{1L} u^{'}_{1R} \chi_2 + \lambda_{2ij} \bar{Q}_{iL} d^{'}_{jR} \chi_2^* + \mathrm{H.c.}, \tag{18}$$

where $i,j \;=\; 2,3$ and χ_2 gets the VEV:

$$\langle \chi_2 \rangle = \begin{pmatrix} 0 \\ 0 \\ w_2 \\ 0 \end{pmatrix}. \tag{19}$$

With the two Higgs fields $\chi_{1,2}$ the gauge symmetry is broken to the standard model, as indicated below:

$$\begin{array}{c} \mathrm{SU}(3)_C \otimes \mathrm{SU}(4)_L \otimes \mathrm{U}(1)_N \\ \downarrow \langle \chi_1 \rangle, \langle \chi_2 \rangle \\ \mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \end{array} \tag{20}$$

where Y is the linear combination of λ_8, λ_{15} and N which annihilates $\langle \chi_1 \rangle$ and $\langle \chi_2 \rangle$ and one can easily check that it is given by:

$$Y = 2N - \frac{1}{\sqrt{3}} \lambda_8 - \frac{2\sqrt{6}}{3} \lambda_{15}, \tag{21}$$

where λ_8 and λ_{15} are diagonal SU(4) generators with $\lambda_8 = diag(1,1,-2,0)/\sqrt{3}$ and $\lambda_{15} = diag(1,1,1,-3)/\sqrt{6}$. One can easily check that Y is numerically identical to the standard model hypercharge.

Electroweak symmetry breaking and the fermion masses are assumed to be due to the VEVs of the Higgs bosons:

$$\rho \sim (1,4,1), \;\; \eta \sim (1,4,0), \;\; S \sim (1,10,0) \tag{22}$$

These Higgs bosons couple to the fermions through the Yukawa Lagrangian:

$$\mathcal{L}^{\rho}_{Yuk} = \lambda_{1a} \bar{Q}_{1L} d_{aR} \rho + \lambda_{ia} \bar{Q}_{iL} u_{aR} \rho^* + \mathrm{H.c.},$$

$$\mathcal{L}^{\eta}_{Yuk} = \lambda^{'}_{1a} \bar{Q}_{1L} u_{aR} \eta + \lambda^{'}_{ia} \bar{Q}_{iL} d_{aR} \eta^* + \mathrm{H.c.},$$

$$\mathcal{L}^{S}_{Yuk} = G_{ab} \bar{f}_{aL} (f_{aR})^c S + \mathrm{H.c.}, \tag{23}$$

where $a,b = 1,2,3$ and $i,j = 2,3$. If ρ gets the VEV:

$$\langle \rho \rangle = \begin{pmatrix} 0 \\ u \\ 0 \\ 0 \end{pmatrix}, \tag{24}$$

two up- and one down-quarks gain mass. If η gets the VEV:

$$\langle\eta\rangle = \begin{pmatrix} v \\ 0 \\ 0 \\ 0 \end{pmatrix}, \tag{25}$$

then the remaining quarks get masses. If S gets the VEV (note that the 10 representation of SU(4) can be represented as a 4×4 symmetric matrix):

$$\langle S\rangle = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{v'}{\sqrt{2}} \\ 0 & 0 & 0 & 0 \\ 0 & \frac{v'}{\sqrt{2}} & 0 & 0 \end{pmatrix}, \tag{26}$$

then all of the leptons get masses. With the VEVs $\langle\rho\rangle, \langle\eta\rangle, \langle S\rangle$ the intermediate electroweak gauge symmetry is spontaneously broken as follows:

$$\begin{array}{cc} cc & \mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \\ & \downarrow <\rho>, <\eta>, <S> \\ & \mathrm{SU}(3)_C \otimes \mathrm{U}(1)_Q \end{array} \tag{27}$$

The electric charge operator has been identified as $Q = I_3 + \frac{Y}{2}$ where I_3 is given by the SU(4) generator diag(1,-1,0,0)/2 and Y is given in eq.(21).

Note that the VEVs of ρ, η and S transform as $Y = 1$, $\mathrm{SU}(2)_L$ doublets (under the $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ subgroup of $\mathrm{SU}(3)_L \otimes \mathrm{U}(1)_N$ which is left unbroken by $\langle\chi_{1,2}\rangle$). For this reason and the fact that Y (eq.(21)) is identical to the standard model hypercharge, it is clear that the model reduces to the standard model with the correct low energy phenomenology provided $\langle\chi_{1,2}\rangle \gg \langle\rho\rangle,\ \langle\eta\rangle,\ \langle S\rangle$.

Note that in the limit $\langle\chi_2\rangle \gg \langle\chi_1\rangle, \langle\rho\rangle, \langle\eta\rangle, \langle S\rangle$, the model reduces to the Pissano-Pleitez model (with right-handed singlet neutrinos) at a energy scale much less than $\langle\chi_1\rangle$. On the other hand, if $\langle\chi_1\rangle \gg \langle\chi_2\rangle, \langle\rho\rangle, \langle\eta\rangle, \langle S\rangle$, the model reduces to the $\mathrm{SU}(3)_C \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_N$ model discussed before (in the present paper), which has ν_R^a in the $\mathrm{SU}(3)_L$ triplets.

Motivated by the Pisano-Pleitez model, we looked for an interesting extensions which can include right-handed neutrinos. We found an alternative $\mathrm{SU}(3)_C \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_N$ model with right-handed neutrinos which is actually simpler than the original Pisano-Pleitez model. We then pointed out the existence of another Pisano-Pleitez type model with right-handed neutrino, which was had the gauge symmetry $\mathrm{SU}(3)_C \otimes \mathrm{SU}(4)_L \otimes \mathrm{U}(1)_N$.

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[3] This gauge group and fermion representations were already discussed by F.Pisano and Tran Anh Tuan, [ICTP preprint IC/93/200; *in* Proc. of the XIVV Encontro National de Fí́sica de Partí́cular e Campos, Caxambu, 1993]. However, they did not consider symmetry breaking, which is done in the present paper.

[4] See for example, P. Langacker and D. London, Phys. Rev. D38, 886 (1988).